Software Performance Engineering
SWEN 549

Spring 2019

Exam 1

Name: ______________________________

- Answer the questions in the spaces provided on the question sheets.
- If you run out of room for an answer, continue on the back of the page.
- Please write legibly.
- On mathematical problems, please show your work. Partial credit will be given if work is shown.
- Please provide at least one decimal place on all answers with a decimal.
- You have the full period to complete this exam.
- If you have a question, please raise your hand and the instructor will come by. Do not walk up to the instructor.
- This portion of the exam has 9 questions and totals to 90 points

1. (10 points) Which of the following were presented as properties of good metrics? Circle all that apply

   A. Consistency

   B. Representation condition

   C. Ease of measurement

   D. Repeatability

   E. Reliability

   F. Robust

   G. Agile

   H. Polysyllabic

   I. Subjective

   J. Linearity

2. (10 points) Suppose we have a hospital handling patients who are arriving at 5 per hour, following a Markovian process. What is the probability that a patient will be served within a half hour?

Solution:

$\lambda = 5$

$x = 0.5$

$F_{15}(0.5) = 1 - e^{-5*0.5} = 91.7\%$

3. (10 points) Suppose we have a librarian handling patrons who are arriving every 18 minutes, following a Markovian process. What is the probability that a patron will have to wait more than half an hour?

> **Solution:**
>
> by minutes...
>
> $1/\lambda = 18$, so $\lambda = 1/18 = 0.05555$
>
> $1 - F_{1/18}(30) = e^{-30/18} = 18.8\%$
>
> or, by hour...
>
> $1/\lambda = 18/60$, so $\lambda = 10/3 = 3.333$ customers per hour
>
> $1 - F_{3.333}(0.5) = e^{-3.333*0.5} = 18.7\%$

4. (10 points) Suppose we want to ensure that 90% of customers at the post office should not have to wait longer than 7 minutes. How many customers per hour must they be able to process, on average, to achieve this goal?

> **Solution:**
>
> **By hours:**
>
> 7 minutes is 7/60 an hour...
>
> $F_\lambda(7/60) \leq 0.90$
>
> $1 - e^{-7*\lambda/60} \leq 0.90$
>
> $1 - 0.90 \leq e^{-7\lambda/60}$
>
> $0.10 \leq e^{-7*\lambda/60}$
>
> $ln(0.10) \leq ln(e^{-7*\lambda/60})$
>
> $ln(0.10) \leq -7 * \lambda/60$
>
> $-60ln(0.10)/7 > \lambda$
>
> $\lambda > 19.74$ customers per hour
>
> **Or by minutes:**
>
> $F_\lambda(7) \leq 0.90$
>
> $1 - e^{-8\lambda} \leq 0.90$
>
> $1 - 0.90 \leq e^{-7\lambda}$
>
> $0.10 \leq e^{-7*\lambda}$
>
> $ln(0.10) \leq -7 * \lambda$
>
> $\lambda > 0.3289$ in customers per minute
>
> Or...
>
> $\lambda > 0.3289 * 60$
>
> $\lambda > 19.74$ customers per hour

5. (10 points) The checkout at a bank has a customer arrival rate of 35 customers per hour, and they have a service rate of 60 customers per hour. What is the mean queue length? Assume both of these random variables are Markovian, and that the entire bank is handled by one queue.

Solution:

$\rho = 35/60 = 58.3\%$

$\frac{0.583}{1-0.583} = 1.4$

6. (10 points) The checkout at a game store has an average of 4 minutes between each customer arrival. They want to have a mean queue length of at most 3 customers. What mean service rate per hour will they need to achieve this?

(Assume both arrival rate and service rate variables are Markovian.)

Solution:

convert to hours initially

$\frac{1}{\lambda} = \frac{4}{60}$, so $\lambda = 60/4 = 15$

$\frac{\rho}{1-\rho} \leq 3$

$\rho \leq 3 - 3\rho$

$4\rho \leq 3$

$\rho \leq 3/4$

$\rho = \frac{\lambda}{\mu}$

$\frac{3}{4} = \frac{15}{\mu}$

$\mu > 20$ customers per hour

Or... keep it all in minutes until the end...

$\frac{1}{\lambda} = 4$, so $\mu = 1/4$ customers / hour

(same logic above)

$\rho \leq 3/4$

$\rho = \frac{\lambda}{\mu}$

$\frac{3}{4} = \frac{0.25}{\mu}$

$3\mu = 1$

$\mu = 0.3333$ customers / minute

Or $0.3333 * 60 = 20$ customers / hour

7. (10 points) The checkout at a cigar shop gets a new customer on average every 12 minutes. You currently have a service rate of 7 customers per hour, but would like to make your mean response time 60% of what it is now. By what percentage will you need to improve your current service rate to achieve this goal?

   (Assume both arrival rate and service rate variables are Markovian.)

   Solution: $\lambda = 5$ per hour $\mu = 7$ per hour

   First find the current response time...

   $\rho = 5/7 = 0.714$

   $\bar{n} = \frac{\rho}{1-\rho} = 2.5$

   In this system, $X = \lambda = 5$ because we are able to process everybody. So then we apply Little's Law:

   $5R = 2.5$ or $R = 0.5$ hours

   But! We want to improve our R, so...

   $0.60R = 0.30$ hours

   $\bar{n} = 0.30 * 5 = 1.5$ is the desired queue length

   $1.5 = \frac{\rho}{1-\rho}$

   $1.5 - 1.5\rho = \rho$

   $1.5 = 2.5\rho$

   $\rho = \frac{1.5}{2.5} = 0.6$ is our desired ρ

   $0.6 = \frac{\lambda}{\mu} = \frac{5}{\mu_{improved}}$

   $\mu_{improved} = \frac{5}{0.6} = 8.\bar{3}$ customers per hour

   Percentage improvement d is $\frac{\mu_{improved}}{\mu_{old}} = \frac{8.333}{7} = 1.19$

   $d = 1.19$ or 19.0%

8. (10 points) Suppose you have a central server system with a CPU, and IO devices of an SSD and HDD. The CPU, SSD, and HDD all have mean service rates of 3 jobs/ms, 2 job/ms, and 0.75 job/ms, respectively. The global throughput of the system is 0.45 jobs/ms. The utilization of the CPU is measured at 40% and the utilization of the SSD is measured at 25%. What is the expected utilization of the HDD?

   Solution:

   We were given μ for each, so convert to S.

   $S_{cpu} = 1/3 = 0.\bar{3}$, $S_{ssd} = 0.5$, $S_{hdd} = 4/3 = 1.\bar{3}$

   Now use $U_i = X_{global} D_i$

   $0.40 = 0.45 D_{cpu}$, so $D_{cpu} = 0.4/0.45 = 0.\bar{8}$

$0.25 = 0.45D_{ssd}$, so $D_{ssd} = 0.25/0.45 = 0.\bar{5}$

Now use $D_i = V_i S_i$

$0.\bar{3} * V_{cpu} = 0.\bar{8}$, so $V_{cpu} = 2.\bar{6}$

$0.5 * V_{ssd} = 0.\bar{5}$, so $V_{ssd} = 1.\bar{1}$

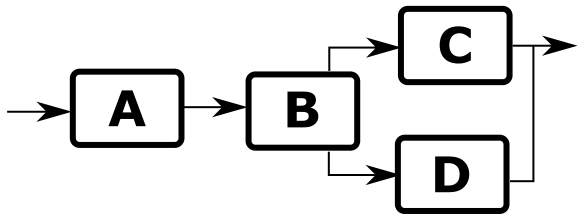
And we know that $V_{cpu} = 1 + V_{ssd} + V_{hdd}$

$2.\bar{6} = 1 + 1.\bar{1} + V_{hdd}$, so $V_{hdd} = 0.\bar{5}$

Use $U_i = X_{global} D_i$, i.e. $U_{hdd} = X_{global} V_{hdd} S_{hdd}$

$U_{hdd} = 0.45 * 0.\bar{5} * 1.\bar{3} = 33.3\%$

9. (10 points) Suppose we have a system similar to the central server system, but with the following structure:



Now suppose that the mean service times for A,B,C, and D are: 0.45, 0.35, 0.7, and 0.3 ms/job respectively. The C queue is visited twice as often as the D queue. The A queue is visited four times as much as the D queue. What is the maximum throughput of this system?

Solution:

Based on the diagram, we can assume that the visit ratios $V_A = V_B$, and that $V_A = 2 + V_C + V_D$

And since we know from the problem that:

$V_C = 2V_D$ and

$V_A = 4V_D$

Then plugging in we get:

$4V_D = 2 + 2V_D + V_D$ $V_D = 2$

so:

$V_C = 4$

$V_B = 8$

$V_A = 8$

Multiply those by the service times to get demands:

$D_A = 4 * 0.5 = 2$

$D_B = 6 * 0.6 = 3.6$

$D_C = 2 * 0.7 = 1.4$

$D_D = 4 * 0.4 = 1.6$

Thus, $D_{max} = 3.6$

So $X_{global} < \frac{1}{3.6} = 0.2\bar{7}$ jobs/ms